Written Exam at the Department of Economics winter 2017-18

Economics of the Environment, Natural Resources and Climate Change

Final reexam

13 February, 2018

(3-hour closed book exam)

This exam question consists of 6 pages in total, including the front page.

NB: If you fall ill during an examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. In this connection, you must complete a form. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

EXERCISE 1. Optimal natural resource extraction with pollution and endogenous exploration

Consider a model of the economy and the environment that uses the following notation:

$Y =$ production of final goods
$K =$ stock of man-made capital (physical and human)
$R =$ input of an exhaustible natural resource (raw material)
$S =$ reserve stock of the natural resource
$E =$ total emission of pollutant
$b =$ emission of pollutant per unit of raw material used in final goods production
$a =$ cost of extracting one unit of the natural resource (measured in units of the final good)
$Q =$ total exploration costs (measured in units of the final good)
$C =$ consumption of final goods
$D =$ discovery of new reserves of the natural resource
$I =$ investment in produced capital
$U =$ lifetime utility of the representative consumer
$u =$ flow of utility from consumption of final goods
$\rho =$ rate of time preference
$t =$ time (treated as a continuous variable)

The pollution from the use of raw materials generates disutility for consumers. The lifetime utility of the representative consumer at time zero is therefore given as

$$U_0 = \int_0^{\infty} \left[u\left(C_t\right) - E_t \right] e^{-\rho t} dt, \qquad u' > 0, \quad u'' < 0, \quad \rho > 0. \tag{1}$$

In the following, all variables except the constant parameters a, b and ρ will be understood to be functions of time, so for convenience we will generally skip the time subscripts.

The production of final goods is given by the production function

$$Y = F\left(K, R\right), \qquad F_K \equiv \frac{\partial F}{\partial K} > 0, \quad F_R \equiv \frac{\partial F}{\partial R} > 0. \tag{2}$$

Pollution is caused by the transformation of the raw material in the process of production. The use of one unit of raw material generates an emission of b units of the pollutant, so total emissions are

$$E = bR, \qquad b \text{ constant.} \tag{3}$$

In each period there is exploration for the discovery of new reserves of the natural resource. The total exploration cost (measured in units of the final good) of discovering the quantity D of new reserves is given by the following exploration cost function:

$$Q = Q\left(D, S\right), \qquad Q_D \equiv \frac{\partial Q}{\partial D} > 0, \quad Q_S \equiv \frac{\partial Q}{\partial S} < 0. \tag{4}$$

The functions F and Q are assumed to have properties which ensure that the first-order conditions for the optimization problems considered below indeed identify maxima.

The total cost of raw material production (measured in units of final goods) is aR, where the constant a is the cost of extracting one unit of the natural resource from the existing stock of reserves. Hence the economy's aggregate resource constraint is

$$Y = C + I + Q + aR, \qquad a \text{ constant}, \quad a > 0. \tag{5}$$

We will abstract from depreciation, so the net investment in man-made capital is equal to the gross investment I. Hence the change over time in the capital stock is

$$\dot{K} \equiv \frac{dK}{dt} = I. \tag{6}$$

The change over time in the natural resource stock is

$$\dot{S} \equiv \frac{dS}{dt} = D - R. \tag{7}$$

The initial stocks of man-made and natural capital (K_0 and S_0) are predetermined.

Our first task is to characterize the first-best optimal allocation of resources that would be chosen by a benevolent social planner who maximizes the utility function (1) subject to the constraints implied by eqs. (2) through (7), taking K_0 and S_0 as given.

Question 1.1: Give a brief motivation for the specification of the exploration cost function (4). (Hint: How do you motivate the assumptions on the signs of the partial derivatives?)

Question 1.2: Show that the current-value Hamiltonian corresponding to the social planner's problem may be written as

$$H = u(C) - bR + \mu\left[F(K,R) - C - Q(D,S) - aR\right] - \lambda R, \tag{8}$$

where μ is the shadow value of K, and λ is the shadow value of S. What are the control variables and what are the state variables in the social planner's optimal control problem?

Question 1.3: Derive the first-order conditions for the solution to the social planner's optimal control problem.

Question 1.4: Show that the first-order conditions for the solution to the social planner's optimal control problem imply that

$$F_R = a + Q_D + MEC, \qquad MEC \equiv \frac{b}{\mu}. \tag{9}$$

Give an economic interpretation of eq. (9) and explain the economic intuition behind it.

Question 1.5: Show that the first-order conditions for the solution to the social planner's problem also imply the following condition for an optimal exploitation of the natural resource, where MEC is defined in (9):

$$\dot{F}_R - Q_S + \rho MEC = \left(F_R - a\right) F_K. \tag{10}$$

Give an economic interpretation of eq. (10) and explain the economic intuition behind it.

We will now consider the resource allocation that will materialize in a market economy with private property and perfect competition in all markets. We start by focusing on the production of raw materials, and we assume that the reserves of natural resources are owned by private mining firms which extract materials from the existing reserve stock and engage in exploration for new reserves. The market value of the representative mining firm at time zero is denoted by V_0^R, and the firm's payout of net dividends in the future period t is denoted by D_t^R. The market value of the firm is the present value of the future net dividends paid out to its owners, that is

$$V_0^R = \int_0^\infty D_t^R e^{-\int_0^t r_s ds} dt, \tag{11}$$

where r is the real market interest rate which may vary over time. The market price of the raw material (measured relative to the price of final goods) is p, and the mining firm must pay an extraction tax τ for each unit of raw material extracted. Both p and τ may also vary over time. The net dividend paid out by the mining firm in period t is therefore given by

$$D_t^R = \left(p_t - a - \tau_t\right) R_t - Q\left(D_t, S_t\right). \tag{12}$$

The mining firm chooses its rate of raw material extraction R and its rate of discovery D so as to maximize its market value (11) subject to (12), taking the price p and the tax rate τ as given, and accounting for the stock-flow relationship (7) between its current rates of extraction and discovery and its remaining reserve stock of the resource.

Question 1.6: Set up the current-value Hamiltonian for the mining firm's optimal control problem and derive the first-order conditions for the solution to its problem (you may denote the shadow price of the reserve stock by λ^R, and for convenience you may skip the time subscripts).

Question 1.7: Show that the mining firm's first-order conditions imply that

$$\dot{p} - \dot{\tau} - Q_S = r\left(p - a - \tau\right). \tag{13}$$

Give an economic interpretation of eq. (13) and explain the economic intuition behind it.

Consider next the representative firm producing the final good Y which we use as our numeraire good, thus setting its price equal to 1. The firm's market value at time zero (V_0^Y) is the present value of its future net dividends,

$$V_0^Y = \int_0^{\infty} D_t^Y e^{-\int_0^t r_s ds} dt, \tag{14}$$

where D_t^Y is the net dividend paid out in the future period t. The output of the final goods firm is given by the production function (2), so the net dividend paid out by the firm is

$$D_t^Y = F\left(K_t, R_t\right) - p_t R_t - I_t. \tag{15}$$

The final goods firm chooses R and I with the purpose of maximizing its market value (14) subject to (15), accounting for the stock-flow relationship (6) between its investment and the change in its capital stock.

Question 1.8: Set up the current-value Hamiltonian for the optimal control problem of the final goods firm (you may denote the shadow price of its capital stock by μ^Y, and for convenience you may skip the time subscripts). Show that the first-order conditions for the solution to the problem of the final goods firm imply that

$$F_R = p, \tag{16}$$

$$F_K = r. \tag{17}$$

Give an economic interpretation of these results.

Question 1.9: Use your findings in Question 1.6, 1.7 and 1.8 to derive an expression for the optimal environmental tax rate τ which will ensure that the use of natural resources in the market economy will satisfy the social optimality conditions (9) and (10). Explain the economic intuition for your result.

EXERCISE 2. The Hartwick Rule and sustainable development

(Note: The questions in this exercise may be answered without any use of math. However, you are welcome to use math to the extent that you find it convenient).

Question 2.1: Explain the Hartwick Rule for natural resource management.

Question 2.2: Explain how the Hartwick Rule relates to the concept of sustainable development.